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Complete Method of Ratios

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This paper outlines two algorithms to compute the average daily access time of a space object by a ground station with restrictions in azimuth, elevation, and range. The first technique, entitled the complete method of ratios, makes extensive use of spherical trigonometry to arrive at a viewing solution whereas the second technique, entitled multiple antennas, uses site topocentric pointing information to determine access time. For the first technique, the original method of ratios is expanded to include viewing restrictions in azimuth and elevation. Simulation results for both methods are documented for a fictitious site restricted to 3500 km range visibility, with seven pairs of azimuth and elevation restrictions. For this study, orbital motion is modeled with J_2 effects, and a viewing solution truth table is created from a 5 s sequential step. Test results include site latitude variations as well as computer run time comparisons to illustrate the execution performance of each technique. The methods presented here are useful tools for mission planners to rapidly evaluate all objects in a space catalog and maintain a current database of objects that are visible to a particular ground station.

Nomenclature

AZ_{HI} = site azimuth upper limit
AZ_j = topocentric azimuth of satellite with respect to the jth site
AZ_{LIM} = site azimuth limit
AZ_{LO} = site azimuth lower limit
e = orbit eccentricity
$\hat{E}_j$ = topocentric unit vector (east) of the jth site
EL_{HI} = site upper elevation restriction
EL_j = topocentric elevation of satellite with respect to the jth site
EL_{LO} = site lower elevation restriction
i = inclination
j = visibility criterion index
j = index of sites on a latitude band
J_2 = second harmonic coefficient (Earth oblateness parameter)
k = number of sites on a latitude band that can see the satellite
L = site latitude
L' = space object latitude
m = number of discrete increments for moving periapsis one cycle
MA = mean anomaly
$\bar{n}$ = anomalistic mean motion
n = number of data points for one orbit
n_0 = mean motion at epoch
$N_{\theta_{MAX}}$ = minimum number of orbit revolutions for complete viewing of latitude band
P = viewing probability
P_{HI} = viewing probability associated with upper elevation restriction
P_{LO} = viewing probability associated with lower elevation restriction
p = semilatus rectum
$\boldsymbol{R}$ = object vector in the fundamental plane
R = object geocentric range
$R_{APOAPSIS}$ = geocentric apoapsis range
R_{CUTOFF} = geocentric cutoff range
R_{MAX} = maximum geocentric range for viewing
R_{MIN} = minimum geocentric range for viewing
$R_{PERIAPSIS}$ = geocentric periapsis range
$\boldsymbol{R}_{SITE}$ = site vector in the fundamental plane
$\hat{S}_j$ = topocentric unit vector (south) of the jth site
$\mathbb{P}$ = orbital time period
x_j = visibility criterion ratio
$\hat{Z}_j$ = topocentric unit vector (zenith) of the jth site
α = latitude difference angle
α_{BEST} = best latitudinal difference angle
α_{MAX} = maximum latitudinal difference angle
γ_1 = first intermediate variable for coverage angle
γ_2 = second intermediate variable for coverage angle
ζ_1 = geocentric vector in equatorial plane aligned with site meridian
ζ_2 = geocentric vector in equatorial plane perpendicular to site meridian
ζ_3 = geocentric vector perpendicular to equatorial plane
θ = complete longitudinal coverage angle of specified latitudinal band
θ_{HI} = longitudinal coverage angle for upper elevation restriction
θ_{LO} = longitudinal coverage angle for lower elevation restriction
θ_{MAX} = maximum longitudinal coverage angle
θ_1 = first longitudinal coverage angle associated with an azimuth limit
θ_2 = second longitudinal coverage angle associated with an azimuth limit
λ = geocentric angle between site and object
$\boldsymbol{\rho}_j$ = topocentric position vector of satellite with respect to the jth site
ρ_j = topocentric range of satellite with respect to the jth site
ρ_{E_j} = east component of topocentric satellite position vector
ρ_{S_j} = south component of topocentric satellite position vector

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ρ_{z_j} = zenith component of topocentric satellite position vector
ρ_{MAX} = site maximum sensor range
ϕ_j = longitude of the jth site
$\dot{\Omega}$ = node rate
$\Omega_\oplus$ = nodal regression angle for one orbit
$\dot{\omega}$ = periapsis rate
$\omega_\oplus$ = Earth rotation rate

Introduction

SPACE operations personnel have witnessed the artificial space population grow at a linear rate of 240 objects per year. International participation in space and the domestic reliance on orbital remote sensing and communications have created a population of artificial Earth satellites totaling 7000 objects. Tables 1 and 2 illustrate this by listing the top five countries with orbiting and decayed satellites, respectively.

The disparity between the tables of orbiting and decayed satellites is primarily from satellite deployment strategies as well as the target orbit of the space shot. Typically, a deep-space launch leaves two to three times more objects in space than does its near-Earth counterpart.

A revised strategy is needed to locate and task satellite tracking sensors with viewing restrictions in range, azimuth, and elevation; this becomes tremendously important for surveillance sites with the added responsibility of missile warning. Each time a site identifies a space object with classical elements that do not match a member of its database, operations personnel must alert Cheyenne Mountain Air Force Base orbital analysts of the uncorrelated contact. If the site's database is not current, this becomes a frequent occurrence.

Tasking a surveillance radar and maintaining its database are key issues to efficiently track space objects. To do so requires a processing algorithm that is both numerically efficient and robust enough to address numerous visibility restrictions. These viewing restrictions range from geopolitical to physical constraints and can sum to irregular regions carved out of the celestial sphere. One possible method to determine average viewing time is to use a step-by-step propagation scheme to simulate a sufficiently large number of passes of a space object vs a station and then record statistics about the angular separation of their Earth-centered inertial position vectors. A similar method involves creating a list of topocentric slant range and azimuth-elevation angles to determine if and when an object becomes visible to a site; a table of these entry/exit times is called a visibility report. Two drawbacks of those methods are lengthy computation time and the dependence on the time rate of change of the argument of periapsis $\dot{\omega}$ to arrive at a reasonable answer. As $\dot{\omega}$ tends to zero, the simulation run time becomes prohibitively long because the accuracy of the visibility report depends on modeling all possible passes of a space object. A geometric solution to determine viewing opportunities can be found in Ref. 1, but this method gives no insight about contact duration for a site with multiple restrictions in range, azimuth, and elevation.

Outlined next are two methods to determine sensor tasking and database maintenance: *the complete method of ratios* and *multiple antennas*. These methods attempt to minimize simulation runtime while maximizing the accuracy of the average daily access time; each technique is discussed in turn.

Table 1 Orbiting satellites

Country	Payloads	Debris	Total
Commonwealth of Independent States	1175	2049	3224
United States	567	2541	3108
European Space Agency	23	120	143
Japan	42	50	92
People's Republic of China	10	81	91

Table 2 Decayed satellites

Country	Payloads	Debris	Total
Commonwealth of Independent States	1561	9249	10,810
United States	598	2851	3413
European Space Agency	3	453	456
People's Republic of China	21	57	78
Japan	9	65	74

Complete Method of Ratios

The concept of ratios was first used by Hayes[2,3] to study the contact between a spaceborne sensor and an Earth target. Davidoff[4] applied this method for ground sites in the Northern Hemisphere with no viewing restrictions. Negron et al.[5] documented an independent derivation for the concept of ratios and refined its use; the technique was successfully applied to determine average daily access times for range-restricted ground stations in either hemisphere for any closed orbit. What follows is an extension of this work, where the range-restricted site is subject to additional viewing constraints in azimuth and elevation; hence the title the complete method of ratios.

The complete method of ratios is based on the assumption that the space object is visible to all sites with latitude L, sensor range ρ_{MAX}, and lower elevation limit EL_{LO} with similar frequency. The Earth is considered spherical for viewing geometry and the propagation of a single orbit. The mean anomaly is chosen as the controlling variable for propagation because a constant step size allows the object to linger over a hemisphere as determined by its orbital eccentricity and periapsis. To save processing time, orbit propagation is only done for one orbit, creating n discrete data points that are stored for further processing. For the cases where Earth oblateness causes apsidal drift, ω is initialized in the Southern Hemisphere and the average viewing time is computed for one orbit. This process is repeated, moving periapsis in m discrete increments until completing one 360-deg cycle. The mean anomaly and periapsis step sizes are designer chosen; for this study n is 300 and m is 8. Advancing periapsis by 45 deg results in symmetry between the site latitude band and the space object, reducing the number of iterations from eight to five for further savings in processing time.

Ratio Equation for Range and Elevation Restrictions

As shown in Fig. 1, the $(\zeta_1, \zeta_2, \zeta_3)$ coordinate system is used to express the geometry between a space object and a site with sensor range ρ_{MAX}. This coordinate system is created such that its principal axis is aligned on the sensor meridian, the equator serves as the fundamental plane, and the origin is at the geocenter. Positioning the space object at maximum sensor range as shown in Fig. 1, the site and object position vectors become

$$R_{SITE} = \cos(L)\hat{\zeta}_1 + \sin(L)\hat{\zeta}_3 \tag{1}$$

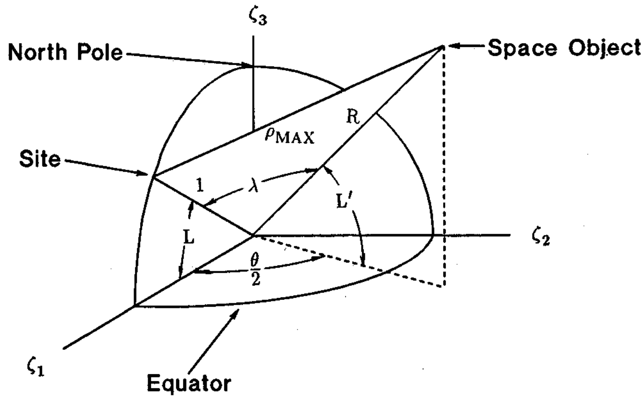

Fig. 1 North/east geometry between a site and space object.

and

$$R = R\cos(L')\cos(\theta/2)\hat{\xi}_1 + R\cos(L')\sin(\theta/2)\hat{\xi}_2 + R\sin(L')\hat{\xi}_3 \tag{2}$$

To take advantage of east/west symmetry, the longitudinal coverage angle θ associated with the site's latitude band is twice the angular measure between $\boldsymbol{R}$ and $\boldsymbol{R}_{\text{SITE}}$ in the fundamental plane; one Earth radii is the unit of length. From the inner product, the cosine of the geocentric angle λ is given by

$$\cos(\lambda) = \frac{\boldsymbol{R}_{\text{SITE}}\cdot\boldsymbol{R}}{R} = \cos(L)\cos(L')\cos(\theta/2) + \sin(L)\sin(L') \tag{3}$$

and the ratio equation becomes

$$\theta = 2\cos^{-1}\left[\frac{\cos(\lambda) - \sin(L)\sin(L')}{\cos(L)\cos(L')}\right] \tag{4}$$

A lower elevation restriction EL_{LO} creates a cone above the site that is topped by a dome at the maximum sensor range; the geocentric range where the two meet is defined as

$$R_{\text{CUTOFF}} = \sqrt{\rho_{\text{MAX}}^2 + 1 + 2\rho_{\text{MAX}}\sin(\text{EL}_{\text{LO}})} \tag{5}$$

From the law of cosines, if a space object enters the dome, then

$$\cos(\lambda) = \frac{R^2 + 1 - \rho_{\text{MAX}}^2}{2R}, \qquad R \geq R_{\text{CUTOFF}} \tag{6}$$

whereas if a space object enters the cone, then

$$\cos(\lambda) = \frac{\cos(\text{EL}_{\text{LO}})^2 + \sin(\text{EL}_{\text{LO}})\sqrt{R^2 - \cos(\text{EL}_{\text{LO}})^2}}{R} \qquad R < R_{\text{CUTOFF}} \tag{7}$$

Because the object is assumed visible to each site with similar frequency, the likelihood of being within range and above the lower elevation restriction becomes

$$P_{\text{LO}}(\text{visibility}) = \theta_{\text{LO}}/2\pi \tag{8}$$

In a similar manner, P_{HI} is found by substituting EL_{HI} for EL_{LO} in Eqs. (5) and (7).

Equation (4) presents numerical difficulties when the absolute value of the inverse cosine argument becomes >1, which can occur frequently for a site with infinite sensor range placed near a pole. To avoid this condition, it is necessary to understand how the argument represents the physical relationship between the site and the space object. If the inverse cosine argument becomes ≥ 1, then the object is completely out of view and θ is 0. If the argument becomes ≤ -1, then the object can be viewed by the entire latitude band and the longitudinal coverage angle becomes 2π.

Incorporating Azimuth Restrictions

The formulation just presented accounts for range and elevation restrictions when computing longitudinal coverage of the site's latitude band. If those results are compared with the coverage associated with restrictions in azimuth only, then any overlap of longitudinal coverage satisfies all restrictions. For nonzero ratio equation results, the longitudinal coverage angles associated with an azimuth limit AZ_{LIM} must be computed; these angles θ_1 and θ_2 are determined using spherical trigonometry as shown in Fig. 2. To do so, intermediate spherical angles γ_1 and γ_2 are computed as

$$\gamma_1 = \sin^{-1}\left[\frac{\sin(\text{AZ}_{\text{LIM}})\cos(L)}{\cos(L')}\right] \tag{9a}$$

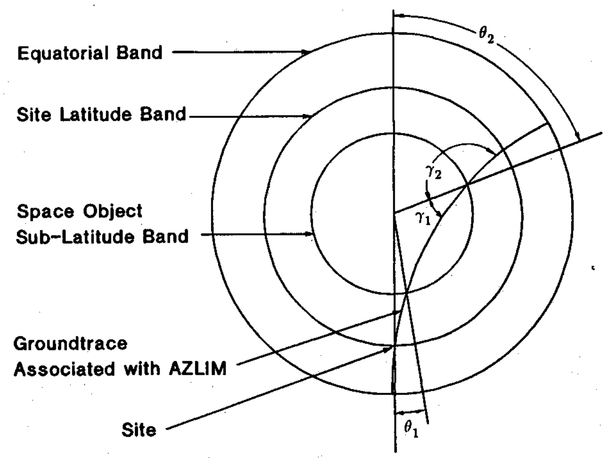

Fig. 2 Azimuth limit geometry as viewed from North Pole.

and

$$\gamma_2 = \pi - \gamma_1 \tag{9b}$$

If the inverse sine of Eq. (9a) does not exist, then full coverage is assumed and the coverage angles are set to π. If L and L' are very close, then the coverage angles are set to 0 and π. Otherwise, the coverage angles are found from

$$\theta_i = 2\tan^{-1}\left[\frac{\sin[(L' - L)/2]}{\cos[(L' + L)/2]\tan[(\gamma_i - \text{AZ}_{\text{LIM}})/2]}\right] \quad (i = 1, 2) \tag{10}$$

where a positive value means the coverage is to the east. Because an easterly restriction may produce a westerly coverage angle and vice versa, rectification of θ_i with the azimuth limit is required; this is accomplished by inverting the sign of θ_i if $\text{AZ}_{\text{LIM}} > \pi$. Finally, a negative coverage angle is set to π, which is the maximum value for coverage. The two resulting angles θ_1 and θ_2 range from 0 to π and represent the longitudinal limits of latitude band coverage in the direction (east or west) of the azimuth limit. These values are then placed in a coverage table and processed as shown in Appendix B.

Visibility Filter

An algorithm to filter space objects that are never visible to a sensor site with latitude L, range ρ_{MAX}, and lower elevation restriction EL_{LO} is needed to reduce processing time. For a site to view a space object, the object must 1) have an elevation angle greater than the site's minimum and 2) be within sensor range. Presented next are equations to prescreen these constraints.

To determine if a space object ever has sufficient latitude for viewing, a maximum latitudinal difference angle,

$$\alpha_{\text{MAX}} = \tan^{-1}\left[\frac{\rho_{\text{MAX}}\cos(\text{EL}_{\text{LO}})}{1 + \rho_{\text{MAX}}\sin(\text{EL}_{\text{LO}})}\right] \tag{11}$$

is compared with the best latitudinal difference angle,

$$\alpha_{\text{BEST}} = \begin{cases} |L| - L'_{\text{MAX}} & |L| > L'_{\text{MAX}} \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

with

$$L'_{\text{MAX}} = \begin{cases} \text{inclination} & \text{direct orbit} \\ \pi - \text{inclination} & \text{indirect orbit} \end{cases} \tag{13}$$

The site will never see the object if $\alpha_{\text{BEST}} > \alpha_{\text{MAX}}$. If the latitudinal difference angle $\alpha = |L - L'|$ is less than α_{MAX}, then the minimum geocentric range for viewing is

$$R_{MIN}(\alpha) = \frac{\cos(EL_{LO})}{\cos(EL_{LO} + \alpha)} \tag{14}$$

and the maximum geocentric range becomes

$$R_{MAX}(\alpha) = \begin{cases} \cos(\alpha) + \sqrt{\rho_{MAX}^2 - \sin^2(\alpha)} & \rho_{MAX}^2 \geq \sin^2(\alpha) \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

Thus, the object is never visible to the site if 1) $\alpha_{BEST} > \alpha_{MAX}$, or 2) $R_{APOAPSIS} < R_{MIN}(\alpha_{BEST})$, or 3) $R_{PERIAPSIS} > R_{MAX}(\alpha_{BEST})$.

Visibility Criterion

As stated earlier, the method of ratios is based on the assumption that a space object is visible to all sites at latitude L with similar frequency. If the satellite ground trace repeats in such a way that sites at certain longitudes on the prescribed latitude band never have a viewing opportunity, then the assumption is violated, an example being a geosynchronous satellite that is only visible to one side of the Earth. To determine this, the following visibility criterion should be applied.

The visibility criterion is violated if a place can be found on the latitude band that does not have a viewing opportunity before the ground trace repeats. Define θ_{MAX} as the maximum value of all discrete values of θ that are available from computing the ratio equation. If θ_{MAX} is 2π, then at some point in its orbit the object is visible to all sites with latitude L and sensor range ρ_{MAX}; hence the visibility criterion is satisfied. For $\theta_{MAX} < 2\pi$ the minimum number of orbital revolutions for complete latitudinal viewing opportunity becomes

$$N_{\theta_{MAX}} = 2\pi/\theta_{MAX} \tag{16}$$

To determine if the ground trace repeats before giving all locations on the latitude band a viewing opportunity, θ_{MAX} is compared with the absolute movement of the ascending node per orbital revolution $\Omega_{\oplus}$, as shown in Fig. 3. This movement is computed as

$$\Omega_{\oplus} = |\omega_{\oplus} - \dot{\Omega}| \mathbb{P} \tag{17}$$

Rounding $\Omega_{\oplus}/\theta_{MAX}$ to the nearest integer gives the minimum number of revolutions of the ascending node about a rotating earth $N_{\Omega_{\oplus}}$ to insure sufficient latitudinal viewing. Numerically, this is determined from the equation

$$x_j = \frac{2\pi j}{\Omega_{\oplus}}, \qquad j = 1, 2, \ldots, N_{\Omega_{\oplus}} \tag{18}$$

where if any x_j is an integer in the range 0 to $N_{\theta_{MAX}}$, exclusive, then the ground trace pattern causes coverage gaps on the latitude band. An algorithm for the visibility criterion is given in Appendix C.

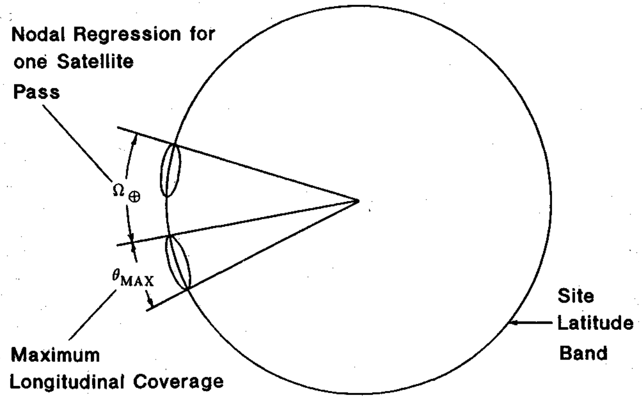

Fig. 3 Visibility criterion geometry as viewed from North Pole.

Multiple Antennas

Outlined next is an alternate method to compute average daily access time between a space object and a site with viewing restrictions in range, azimuth, and elevation. The technique involves computing topocentric pointing information for numerous sites on the latitude band that are evenly spaced in longitude, with each site having common viewing restrictions; the same assumptions that applied to the complete method of ratios apply here. Because the concept of multiple antennas is a numerical method that uses topocentric pointing information to determine visibility for a specified satellite position, it differs significantly from the analytic, geocentric method of ratios. For the multiple antennas simulation, the satellite is advanced in its orbital plane at 1.2 deg in mean anomaly and periapsis rotated at 45-deg increments.

This method simulates sites evenly spaced in longitude that populate the L latitude band, computes pointing information for each site, and sums those sites that have visibility. To enhance computational efficiency, the topocentric position vectors $\boldsymbol{\rho}_j$ are computed using projections instead of rotation matrices. This is done by computing the satellite vector in the *IJK* frame in the form

$$\boldsymbol{R}_{SAT} = [R_I, R_J, R_K]^T \tag{19}$$

The site *SEZ* vectors are defined in the *IJK* system as

$$\hat{S}_j = [\cos(\phi_j)\sin(L), \sin(\phi_j)\sin(L), -\cos(L)]^T \tag{20a}$$

$$\hat{E}_j = [-\sin(\phi_j), \cos(\phi_j), 0]^T \tag{20b}$$

$$\hat{Z}_j = [\cos(\phi_j)\cos(L), \sin(\phi_j)\cos(L), \sin(L)]^T \tag{20c}$$

where complete latitude band coverage is accomplished by rotating the site longitude ϕ_j in evenly spaced increments about $\hat{K}$; for this study $j = 1, \ldots, 1000$ and the *SEZ* vectors are computed only once and stored in tables. The incremental components of $\boldsymbol{\rho}_j$ in the *SEZ* system become

$$\rho_{S_j} = \boldsymbol{R}_{SAT} \cdot \hat{S}_j \tag{21a}$$

$$\rho_{E_j} = \boldsymbol{R}_{SAT} \cdot \hat{E}_j \tag{21b}$$

$$\rho_{Z_j} = \boldsymbol{R}_{SAT} \cdot \hat{Z}_j - 1 \tag{21c}$$

Range, azimuth, and elevation are computed as

$$\rho_j = \sqrt{\rho_{S_j}^2 + \rho_{E_j}^2 + \rho_{Z_j}^2} \tag{22a}$$

$$AZ_j = ATAN2(\rho_{E_j}, -\rho_{S_j}) \tag{22b}$$

$$EL_j = \sin^{-1}(\rho_{Z_j}/\rho_j) \tag{22c}$$

The preceding quantities are then tested against the viewing restrictions, and a summation index k is incremented whenever all constraints are met (i.e., whenever site j sees the space object).

This process is similar to the preceding method where one seeks the ratio of sites with visibility to the total number of sites on the latitude band to arrive at $P(\text{visibility}) = k/1000$. Polling all sites on the latitude band to determine longitudinal coverage is more direct than the ratios method, but carries a greater computational burden. The multiple antennas solution is also useful for validating the complete method of ratios because both determine the same longitudinal coverage for a specified satellite position.

Simulation Results

The complete method of ratios and multiple antennas solution sets are compared with a visibility report created from a 5 s sequential step along the orbit, with viewing constraints checked at each integration step. To generate this data, which serves as the truth, the object and ground station were initialized at the vernal equinox and their relative positions evaluated for 5 years. Satellite motion is modeled using first-order secular perturbations that are caused by J_2 and are listed next[6]:

$$\bar{n} = n_0\left[1 + \frac{3}{2}J_2\frac{\sqrt{1-e^2}}{p^2}\left(1 - \frac{3}{2}\sin^2 i\right)\right] \tag{23}$$

$$\dot{\Omega} = -\left(\frac{3}{2}\frac{J_2}{p^2}\cos i\right)\bar{n} \tag{24}$$

$$\dot{\omega} = \left[\frac{3}{2}\frac{J_2}{p^2}\left(2 - \frac{5}{2}\sin^2 i\right)\right]\bar{n} \tag{25}$$

To conduct the simulation, classical orbital elements for a circular and an eccentric orbit were obtained from the United States Space Command space object catalog and are listed in Table 3. To demonstrate the versatility of both the complete method of ratios and multiple antennas, a fictitious site was created such that its viewing constraints are perhaps more restrictive than one finds in the field. The site, modeled similar to a phased-array radar, has a limited field of view that contains numerous obstructions; the viewing window ranges from 50 to 225 deg in azimuth and 0 to 85 deg in elevation, with visibility restricted to a 3500 km range. The azimuth limits were chosen to test the ratios formulation and programming; an oversight in either area would most likely produce singularities or wrong answers when transitioning quadrants. In addition to the viewing window, the site also has six pairs of exclusive azimuth-elevation constraints listed in Table 4; as shown in Fig. 4, those subconstraints form the letters "HI!" when viewed from the site.

Because satellites are deployed with only finite precision, the criteria to declare an orbit inertially or Earth fixed are designer chosen. For this study, periapsis drift is considered negligible when inclination is within 2 deg of a critical inclination and eccentricity is greater than 0.1; this reduces processing time by eliminating the need to advance periapsis in m increments, as is also the case for circular orbits. An orbit is considered geostationary if it has an inclination less than 10 deg, an eccentricity less than 0.001, and a period within 15 s of the sidereal period; this violates the visibility criterion, and the method of ratios should not be used. Sorting of the United States Space Command catalog indicates that 5% of the cataloged objects are critically inclined, 74% are circular ($e < 0.05$), and 2% are geostationary.

Table 3 Classical orbital elements ($\omega = \Omega = MA = 0$ deg)

Satellite	N, rev/solar day	e	i, deg
1	15.58702863	0.0018614	51.6006
2	8.72593328	0.1338821	87.2805

Table 4 Exclusive visibility restrictions

Restriction	Clockwise azimuth limits, deg	Elevation limits, deg
A	125–127	38–52
B	127–131	44–46
C	131–133	38–52
D	137–139	38–52
E	143–145	42–52
F	143–145	38–40

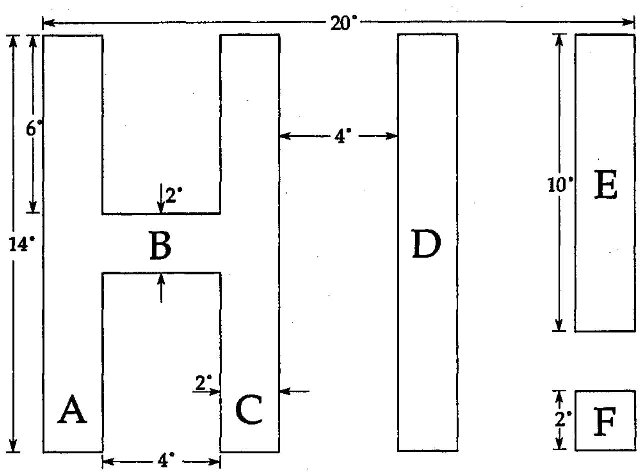

Fig. 4 Exclusive azimuth and elevation restrictions.

Table 5 Satellite 1 average daily access time (min/day)

Site latitude, °	Ratio equation	Multiple antennas	Truth, 5 yr
88	0	0	0
44	30.5	30.5	30.5
0	17.0	16.8	16.8
−44	29.4	29.4	29.4
−88	0	0	0

Table 6 Satellite 2 average daily access time (min/day)

Site latitude, °	Ratio equation	Multiple antennas	Truth, 5 yr
88	23.3	23.3	21.4
44	4.9	4.9	4.8
0	4.0	3.9	4.0
−44	6.0	5.9	6.5
−88	28.4	28.4	29.9

Tables 5 and 6 show that the results from the complete method of ratios and the multiple antennas simulation are nearly identical, as they should be. The variances in the Table 6 truth model reveal that 5 years were not always sufficient to reach an average; a 20-year run for the 88-deg site produced an average visibility of 22.8 min with slow, cycling growth.

The simulations were run on a MicroVAX 3600; the complete method of ratios required 4.43 CPU s, the multiple antennas method ran in 289.27 CPU s, and the 5-year truth simulation took 11.5 CPU h. The spherical trigonometry and table generation of the complete method of ratios yields a 65-fold savings in run time over the more direct multiple antennas method.

Closing Remarks

This paper presents two algorithms to rapidly determine the average daily access time of a space object by a ground station with viewing restrictions in range, azimuth, and elevation. The methods outlined are easily implemented on a personal computer, with the complete method of ratios quickly processing thousands of objects spanning the range of closed orbits against any site latitude. These methods are also useful in performing parametric studies, such as identifying the payoff of various upgrade strategies to a tracking station network; this is done by varying site latitude and sensor constraints using average daily access time as the performance index.[5] Whether attempting to quantify the cost per unit increase in ground system performance or allocating funds to improve or maintain a network of stations, the merits of these tools are easily recognized. Also, these processing algorithms are useful in identifying the optimal site latitude to track a class of orbits by holding viewing constraints fixed while varying the site latitude. Depending on the satellite viewing priority, these tools can help mission designers position a ground station to maximize the mean cumulative visibility or reposition mobile tracking terminals to maximize average daily contact once a space vehicle has maneuvered. These methods can also be used in selecting orbital characteristics to maximize contact with a viewing-restricted site, supporting space mission designs for highly maneuverable vehicles.

Appendix A: Algorithm for the Complete Method of Ratios

1) Set L equal to the site latitude.
2) Set L' equal to the object sublatitude.

3) Set R equal to object geocentric range.
4) Perform visibility prefiltering for the lower elevation:

$$\alpha = |L - L'|$$

If $\alpha > \alpha_{MAX}$, then stop; the object is below site elevation angle, $\theta = 0$.
5) Perform visibility prefiltering for the minimum range using α from step 4. If $R < R_{MIN}(\alpha)$, then stop; the object is below range, $\theta = 0$.
6) Perform visibility prefiltering for the maximum range using α from step 4. If $R > R_{MAX}(\alpha)$ then stop; the object is beyond range, $\theta = 0$.
7) Use the algorithm in Appendix B to compute θ and P(visibility).

Appendix B: Algorithm for Determining Overlapping Azimuth and Ratio Constraints

1) Compute θ_{LO} from the ratio equation using EL_{LO}; if $\theta_{LO} = 0$, then stop, the site cannot see the satellite. Set up a paired table of (θ, associated viewing flags) as $(-\theta_{LO}/2, 1)$ and $(\theta_{LO}/2, -1)$.
2) Compute θ_{HI} from the ratio equation using EL_{HI}. Set up a paired table as $(-\theta_{HI}/2, 1)$ and $(\theta_{HI}/2, -1)$
3) Compute θ_1 and θ_2 from the coverage equation using AZ_{LO}. Set up a paired table as follows:
If $L' \geq L$, then
If azimuth restriction is clockwise,
If $AZ_{LO} \leq \pi$, then $(-\pi, 1)$, $(0, -1)$, $(\theta_1, 1)$, and $(\theta_2, -1)$
If $AZ_{LO} > \pi$, then $(-\pi, 1)$, $(0, -1)$, $(-\theta_1, 1)$, and $(-\theta_2, -1)$
If azimuth restriction is counterclockwise,
If $AZ_{LO} \leq \pi$, then $(0, 1)$, $(\theta_1, -1)$, and $(\theta_2, 1)$
If $AZ_{LO} > \pi$, then $(0, 1)$, $(-\theta_1, -1)$, and $(-\theta_2, 1)$
If $L' < L$, then
If azimuth restriction is clockwise,
If $AZ_{LO} \leq \pi$, then $(-\pi, 1)$, $(\theta_1, -1)$, and $(\theta_2, 1)$
If $AZ_{LO} > \pi$, then $(-\theta_1, -1)$, and $(-\theta_2, 1)$
If azimuth restriction is counterclockwise,
If $AZ_{LO} \leq \pi$, then $(\theta_1, 1)$ and $(\theta_2, -1)$
If $AZ_{LO} > \pi$, then $(-\pi, 1)$, $(-\theta_1, 1)$ and $(-\theta_2, -1)$
4) Compute θ_1 and θ_2 from the coverage equation using AZ_{HI}. Set up a paired table as follows:
If $L' \geq L$, then
If azimuth restriction is clockwise,
If $AZ_{HI} \leq \pi$, then $(0, 1)$, $(\theta_1, -1)$, and $(\theta_2, 1)$
If $AZ_{HI} > \pi$, then $(0, 1)$, $(-\theta_1, -1)$, and $(-\theta_2, 1)$
If azimuth restriction is counterclockwise,
If $AZ_{HI} \leq \pi$, then $(-\pi, 1)$, $(0, -1)$, $(\theta_1, 1)$, and $(\theta_2, -1)$
If $AZ_{HI} > \pi$ then $(-\pi, 1)$, $(0, -1)$, $(-\theta_1, 1)$, and $(-\theta_2, -1)$
If $L' < L$, then
If azimuth restriction is clockwise,
If $AZ_{HI} \leq \pi$, then $(\theta_1, 1)$ and $(\theta_2, -1)$
If $AZ_{HI} > \pi$, then $(-\pi, 1)$, $(-\theta_1, 1)$ and $(-\theta_2, -1)$
If azimuth restriction is counterclockwise,
If $AZ_{HI} \leq \pi$, then $(-\pi, 1)$, $(\theta_1, -1)$, and $(\theta_2, 1)$
If $AZ_{HI} > \pi$, then $(-\theta_1, -1)$, and $(-\theta_2, 1)$
5) Combine tables from steps 1, 3, and 4. Reorder from least angle to greatest. Set $\theta_{LO} = 0$.
6) Set COUNTER = 0 and increment with flag values from step 5 table:
If the azimuth limits are clockwise and COUNTER = 3, then
$\theta_{LO} = \theta_{LO}$ − present table angle + next table angle
If the azimuth limits are counterclockwise and COUNTER = 2, then
$\theta_{LO} = \theta_{LO}$ − present table angle + next table angle.
7) Combine tables from steps 2, 3, and 4. Reorder from least angle to greatest. Set $\theta_{HI} = 0$.
8) Set COUNTER = 0 and increment with flag values from step 7 table:
If the azimuth limits are clockwise and COUNTER = 3, then
$\theta_{HI} = \theta_{HI}$ − present table angle + next table angle
If the azimuth limits are counterclockwise and COUNTER = 2, then
$\theta_{HI} = \theta_{HI}$ − present table angle + next table angle
9) $P(\text{visibility}) = (\theta_{LO} - \theta_{HI})/2\pi$.

Appendix C: Algorithm for the Visibility Criterion

1) Get θ_{MAX} from the ratio equation.
2) If θ_{MAX} is 2π, then stop; criterion is satisfied.
3) Compute $\Omega_{\oplus}$.
4) Compute $N_{\theta_{MAX}}$ and $N_{\Omega_{\oplus}}$.
5) Compute and evaluate x_j for an integer value, letting j range from 1 to $N_{\Omega_{\oplus}}$.

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